

A WIN-WIN STRATEGY FOR AN INTEGRATED VENDOR-BUYER DETERIORATING INVENTORY SYSTEM

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Abstract. An integrated approach considering the view of both the buyer and the vendor is discussed in this study. It can be shown numerically that the integrated approach results in an impressive cost reduction when compared with an independent decision approach by the buyer. Although the integrated total cost decreases, the buyer's cost increases due to larger orders. To entice the buyer to accept larger order quantity, a permissible delay in payment is offered by the vendor to the buyer. A negotiation factor is also incorporated to share the benefits.

Key words: integration, deterioration, permissible delay in payment

1. Introduction

In general, a retailer has the privilege to decide on the lot size when an order is made. The optimal decision derived solely from the perspective of the buyer may not be the most economical one for the vendor. However, if both of their perspectives are taken into account, a joint policy can be achieved. The idea of vendor-buyer integration has been studied in the sixties by Clark and Scarf [2]. Banerjee [1] derived a joint economic lot size with finite production rate. Goyal [4] extended Banerjee's model by relaxing the lot-for-lot production assumption.

Deterioration is defined as decay, spoilage, evaporation, and loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one. IC chip, blood, fish, strawberries, alcohol, gasoline, radioactive chemicals and grain products are examples of deteriorating commodities. Several researchers have studied deteriorating inventory in recent decades. Ghare and Schrader [3] were the first authors to consider on-going

deterioration of inventory. Other authors such as Raafat et al. [7] assumed either instantaneous or finite production with different assumptions on the pattern of deterioration.

Kingsman [5] analyzed the relationship between the inventory cost and the payment rules. Mandal and Phaujdar [6] developed the EOQ model with permissible delay in payment.

In this study, an integrated vendor-buyer inventory system for deteriorating item is developed. A negotiation factor is used to balance the benefit, and a permissible delay in payment is offered to the buyer to make the cooperation relationship more realistic and mutual beneficial.

2. Mathematical Modeling and Analysis

The mathematical model is developed on the basis of the following assumptions:

- (a) A system of single-vendor and single-buyer is considered.
- (b) The deterioration rate is constant and proportional to on-hand stock.
- (c) The demand rate is deterministic and known.
- (d) The order replenishment rate is instantaneous.
- (e) Permissible delay in payment is used to entice the buyer to cooperate in the integrated inventory system.

The following notation is used:

θ	Deterioration rate
T	Vendor's replenishment time interval
T_b	Buyer's replenishment time interval
n	Buyer's order times per T
d	Demand rate
C_v (C_b)	Vendor's (buyer's) unit cost
$I_v(t)$	Vendor-buyer combined inventory level
$I_b(t)$	Buyer's inventory level
C_{v1} (C_{b1})	Vendor's (buyer's) ordering cost
C_{v2} (C_{b2})	Vendor's (buyer's) annual holding cost per unit
TC_v (TC_b)	Vendor's (buyer's) annual total cost
TC	Annual total cost for both vendor and buyer
Δt	Permissible delay in payment offered by the vendor to the buyer
R	Continuous interest rate

If the deterioration rate is proportional to on-hand stock, the inventory system at the vendor or the buyer with constant demand rate is represented by the following differential equations:

$$\frac{dI_b(t)}{dt} + \theta I_b(t) = -d, \quad 0 \leq t \leq \frac{T}{n} \quad (2.1)$$

and

$$\frac{dI_\nu(t)}{dt} + \theta I_\nu(t) = -d, \quad 0 \leq t \leq T. \tag{2.2}$$

The boundary conditions are $I_b(t = \frac{T}{n}) = 0$ and $I_\nu(t = T) = 0$. The solutions of the differential equations are:

$$I_b(t) = \frac{d}{\theta} \frac{e^{\frac{\theta T}{n}} - e^{\theta t}}{e^{\theta t}}, \quad 0 \leq t \leq \frac{T}{n}, \tag{2.3}$$

$$I_\nu(t) = \frac{d}{\theta} \frac{e^{\theta T} - e^{\theta t}}{e^{\theta t}}, \quad 0 \leq t \leq T. \tag{2.4}$$

The lot sizes for the buyer and the vendor are $I_{mb} = \frac{d}{\theta}(e^{\frac{\theta T}{n}} - 1)$ and $I_{m\nu} = \frac{d}{\theta}(e^{\theta T} - 1)$ respectively.

In the time period T , the total holding cost for the buyer is $C_{b2}n \int_0^{\frac{T}{n}} I_b(t)dt$. The actual vendor average inventory level in the integrated two-echelon inventory model is the difference between the vendor-buyer combined average inventory level and the buyer average inventory level. The actual vendor holding cost in time T is expressed as:

$$C_{\nu 2} \left[\int_0^T I_\nu(t)dt - n \int_0^{\frac{T}{n}} I_b(t)dt \right]. \tag{2.5}$$

In the time period T , the deterioration cost for the buyer and the vendor are $C_b n \left[I_{mb} - \frac{dT}{n} \right]$ and $C_\nu \left[I_{m\nu} - I_{mb} \right]$ respectively.

In the time period T , the ordering costs for the buyer and the vendor are $C_{b1}n$ and $C_{\nu 1}$ respectively. The annual buyer's total cost is the cycle time total cost divided by T .

$$TC_b = \frac{C_{b2}n}{T} \int_0^{\frac{T}{n}} I_b(t)dt + \frac{C_b n}{T} \left[I_{mb} - \frac{dT}{n} \right] \frac{C_{b1}n}{T}. \tag{2.6}$$

The annual vendor's total cost is the cycle time total cost divided by T .

$$TC_\nu = \frac{C_{\nu 2}}{T} \left[\int_0^T I_\nu(t)dt - n \int_0^{\frac{T}{n}} I_b(t)dt \right] + \frac{C_\nu}{T} \left[I_{m\nu} - nI_{mb} \right] + \frac{C_{\nu 1}}{T}. \tag{2.7}$$

The annual integrated total cost is the sum of TC_b and TC_ν . Since $T_b = \frac{T}{n}$, TC is a function of two variables: T_b and n .

3. Solution Procedure

For the case without considering integration, the buyer and the vendor make strategic decision independently. In buyer market, the first step is for the

buyer to minimize TC_b by deriving T_b from solving the following equation $\frac{\partial TC_b}{\partial T_b} = 0$. The second step is for the vendor to minimize TC_v by deriving the replenishment times per cycle time. Since the replenishment time is a discrete integer, it must satisfy the following condition:

$$TC_v(n-1) \geq TC_v(n) \leq TC_v(n+1). \quad (3.1)$$

The total cost without considering integration, $TC^\#$ is expressed as

$$TC^\# = \min_n \left[\left(\min_n TC_b \right) + TC_v \right]. \quad (3.2)$$

For the case considering integration, the total cost is optimized jointly rather than independently. The optimal values of T_b and n must satisfy the following conditions simultaneously:

$$\frac{\partial TC}{\partial T_b} = 0 \quad (3.3)$$

and

$$TC(n-1) \geq TC(n) \leq TC(n+1). \quad (3.4)$$

The total cost considering integration, TC^* is expressed as

$$TC^* = \min_{T_b, n} (TC_b + TC_v). \quad (3.5)$$

Since (3.4) is less than (3.2), the total cost saving, S_{int} is defined as $S_{int} = TC^\# - TC^*$. Let the buyer's cost saving, S_b be defined as $S_b = \alpha S_{int}$, where α is the negotiation factor. When the negotiation factor equals one, it means all saving is given to the buyer; when the negotiation factor is 0.5, it implies that the total cost saving is equally distributed between the vendor and the buyer. If the negotiation factor is zero, all saving is given to the vendor. The present value of unit cost after a time interval Δt is $e^{-R\Delta t}$. Solving the following equation can derive the buyer's permissible delay in payment:

$$dC_b(1 - e^{-R\Delta t}) = S_b, \quad (3.6)$$

where R is the continuous interest rate

The close form solution for Δt is

$$\Delta t = \frac{1}{R} \ln \left[\frac{C_r d}{C_r d - \alpha(TC^\# - TC^*)} \right]. \quad (3.7)$$

4. Numerical Example and Sensitivity Analysis

The preceding theory can be illustrated by the following numerical example. The annual demand is 40,000 units. The unit costs for the vendor and the buyer are \$10 and \$12 respectively. The ordering cost for the vendor and the buyer are \$3,000 and \$600 respectively. The annual percentage of holding cost for the vendor and the buyer are 10% and 11% per unit. The annual

Table 1. The optimal solution with and without considering integration.

Cases	Without integration	With integration
n	3	1
T_b	0.1087	0.2649
T	0.3261	0.2649
TC_b	\$11,018	\$15,735
TC_v	\$18,023	\$11,325
TC	\$29,041	\$27,060
$PICR$	–	6.82%
Δt (years)	–	0.06887

Table 2. Sensitivity analysis of deterioration rate.

θ	0.06	0.08	{0.10}	0.12	0.14
$TC^\#$	25.991	27.557	29.041	30.455	31.810
TC^*	24.311	25.721	27.060	28.337	29.561
$PICR$	6.46%	6.66%	6.82%	6.95%	7.07%
Δt	0.05839	0.06382	0.06887	0.07361	0.07817

Table 3. Sensitivity analysis of demand rate.

d	24.000	32.000	{40.000}	48.000	56.000
$TC^\#$	22.529	25.991	29.041	31.799	34.334
TC^*	20.987	24.216	27.060	29.631	31.995
$PICR$	6.84%	6.83%	6.82%	6.82%	6.81%
Δt	0.08934	0.07715	0.06887	0.06279	0.05805

Table 4. Sensitivity analysis of buyer’s holding cost.

C_{b2}	0.066	0.088	{0.11}	0.132	0.154
$TC^\#$	25.931	27.530	29.041	30.477	31.848
TC^*	24.072	25.609	27.060	28.436	29.748
$PICR$	7.17%	6.98%	6.82%	6.70%	6.59%
Δt	0.06462	0.06675	0.06887	0.07095	0.07300

deterioration rate is 10%. The interest rate and the negotiation factor are assumed to be 3% and 0.5 respectively.

By applying the solution procedure from Section 3, the solution comparing the case with integration to the case without integration is given in Table 1. The buyer’s cost and the replenishment interval increase, when integration is considered. The vendor benefits \$6,698, while the buyer losses \$4,717. Therefore, the buyer will be hesitant to support the integration process. To entice the buyer to cooperate, the vendor offers the buyer a permissible delay in

payment of 0.06887 years with equally distributed benefit. The percentage of integrated total cost reduction (*PICR*) is defined as $PICR = \frac{TC^\# - TC^*}{TC^\#}$. The value of *PICR* is 6.82%.

Sensitivity analysis is carried out and given in Table 2 to 4. When the deterioration rate increases (and the demand rate decreases), the permissible delay in payment increase. Therefore, the greater the deterioration rate or the less the demand rate, there is greater need to consider the integration process. The permissible delay in payment increases when the buyer's holding cost increases.

5. Conclusion Remark

In this paper, we develop a mathematical model for deteriorating item to derive an optimal ordering policy in the integrated vendor-buyer inventory system. It is shown that the optimal policy using the integrated approach has resulted in a lower joint cost. However, the buyer's cost increases when the integrated approach is used. To motivate the buyer's cooperation, an incentive system in the form of credit term to the buyer is incorporated into the system. In this analysis, we show that it is more significant to consider the integration process and the permissible delay in payment when deterioration rate and the holding rate are higher.

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