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APPLICATIONS OF CFD AND PCA METHODS FOR GEOMETRY RECONSTRUCTION OF 3D OBJECTS

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Abstract. This article presents method of reconstruction of 3D objects from RTG images and statistical knowledge from database[5]. Resulting 3D object is CAD system compatible. Authors apply Principal Component Analysis [3] which decomposes the population of 3D objects into mean geometry and individual features (empirical modes). The greatest advantage of this method is the possibility of changes of geometry by manipulation of the coefficient values and creation of new 3D objects not existing in database. To achieve the same FEM mesh for all objects in database, the modified fluid registration was used [2]. It applies (using source term in Navier-Stokes equation) the base FEM grid onto geometry of the newly added objects. To validate the developed method, reconstruction of human vertebrae and error estimation was done. In this paper the theoretical basis and results of numerical experiment are discussed.

Key words: 3D reconstruction, CAD, DRR (Digitally Reconstructed Radiograph), PCA (Principal Component Analysis), RTG, registration, solid model

1. Introduction

Nowadays, many engineering CAD technologies have an application not only in mechanics but also in different disciplines like biomechanics, bioengineering, etc. This interdisciplinary research takes advantage of reverse engineering, 3D modelling and simulation, FEM analysis and is equipped with rapid prototyping and NC machines.

The acquisition and processing of 3D models with complicated shapes becomes the important issue in applications mentioned above. The 3D virtual models have numerous applications such as visualization, medical diagnostics (virtual endoscopes), pre-surgical planning, FEM analysis, CNC machining, Rapid Prototyping, etc. New techniques often arise by combination of the known methods from different disciplines into completely new application. In this work the PCA and fluid flow dynamics were used to reconstruct CAD models.

2. Three-Dimensional Modal Shape Description

For reconstruction of the 3D geometry, "low-dimensional" reconstruction based on Principal Component Analysis (PCA, also known as Karhunen-Loeve Decomposition, POD) can be used [1]. PCA provides a "relevant" set of basis functions, which allow identification of a low-dimensional subspace [3].

Using KL expansion, a given statistical process (data can be correlated) is described with the minimum number of uncorrelated modes (principal components). The resulting coordinate system (defined by the eigenfunctions of the correlation matrix) is optimal in the sense that the mean-square error resulting from a finite representation of the process is minimized.

In example presented below, the program used in CFD (Computational Fluid Dynamics) modal analysis is adapted. The shape of the every object is represented in the data base as the 3D FEM grid. Each FEM grid is described by vector (2.1):

$$S_i = [s_{i1}, s_{i2}, ..., s_{iN}]^T, \quad i = 1, 2, ..., M, \tag{2.1}$$

where $s_{ij} = (x, y, z)$ describes coordinates of the nodes (FEM grid) in Cartesian coordinates system, M is the number of the objects which are in data base, N is the number of the FEM nodes of every single object.

After that the mean shape $\bar{S}$ and covariance matrix C are computed (2.2):

$$\bar{S} = \frac{1}{M}\sum_{i=1}^{M} S_i, \quad C = \frac{1}{M}\sum_{i=1}^{M} \tilde{S}_i \tilde{S}_i^T. \tag{2.2}$$

The differences between mean and object that is in data base are described by the deformation vector $\tilde{S}_i = S_i - \bar{S}$. The statistical analysis of the deformation vectors gives us the information about the empirical modes. Modes represent the features: geometrical (shape), physical (density) and others like displacement and rotation of the object.

Only few first modes carry most information, therefore each original object S_i is reconstructed by using some K principal components (2.3):

$$S_i = \bar{S} + \sum_{k=1}^{K} a_{ki}\Psi_k, \quad i = 1, 2, ..., M, \tag{2.3}$$

where Ψ_k is an eigenvector representing the orthogonal mode (the feature computed from data base), a_{ik} is coefficient of eigenvector.

The example of the low dimensional reconstruction for three different values of the coefficients for the first mode is presented on the Fig. 1. For $a_{ki} = 0$ we obtain mean value, for different values we get new variants of vertebrae shapes.

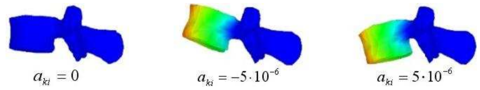

Figure 1. The visualization of the reconstruction for different coefficient values.

3. Viscous Fluid Registration

The Principal Component Analysis requires the same topology of the FEM mesh (the same number of nodes, matrix connection, etc.) for all objects. To achieve this, every new object added to database, must be registered.

The goal of registration is to apply the base grid onto geometry of the new objects. For this registration the modified Navier-Stokes equation in penalty function formulation ([2]) is used (3.1):

$$\underbrace{\dot{V}_i + V_{i,j}V_j - \frac{1}{Re}V_{i,jj} + \frac{\varepsilon - \lambda}{\rho}V_{j,ji}}_{\text{existing numerical code [4]}} + \underbrace{(f-g)f_{,i}}_{\text{source segment [2]}} = 0, \tag{3.1}$$

where ρ is fluid density, V_i velocity component, Re Reynolds number, λ bulk viscosity. In this application parameters ε and λ are used to control the fluid compressibility. The object is described by the FEM grid. The displacements of the nodes are computed from integration of the velocity field.

Every base and new model is represented by several cross-sections with the same transfer area (Fig. 2). The transfer area is needed for topology conservation.

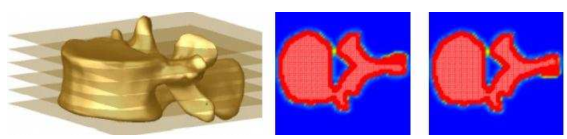

Figure 2. Cross sections of the vertebra (left side): base vertebra, input vertebra (dark gray - the transfer area).

Source term is calculated from the difference of the two images in grey scale (3.1), where f and g are areas in gray-scale: f is base image (base vertebra), g is target image (input vertebra). The parameters of the flow are the same as for the compressible fluid with very viscosity.

Computed flow field provides information about translations of the nodes (FEM grid) in both sections. After computation we obtain dislocation of nodes of the base grid onto new geometry.

This procedure is repeated for all cross-sections and when is done, there is possibility to apply base FEM grid to new geometry of the vertebra being added to the database (Fig. 3).

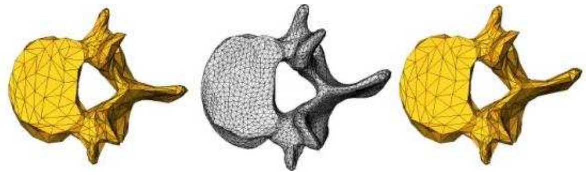

Figure 3. FEM grid deformation: base vertebra (left side), input vertebra (middle), base FEM grid on geometry of the new objects (right side).

4. Numerical Experiment

Algorithm of the method is the following (Fig. 4). Searched three-dimensional objects are represented by the set of RTG images (minimum two images from different directions). These RTG images are compared with DRR - Digitally Reconstructed Radiographs ([5]) from database.

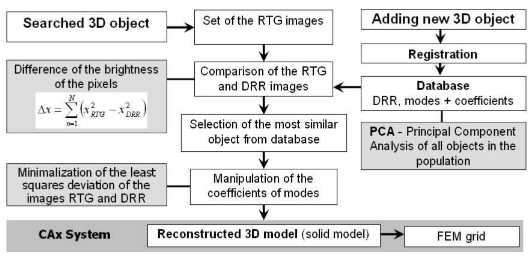

Figure 4. Algorithm of the method.

Database includes DRR images and set of the modes and coefficients. Modes and coefficients are received from Principal Component Analysis. After comparison of images, the most similar objects from database are selected. In the next step we manipulate the coefficients of modes as long as the minimization of the mean square deviation of the images RTG and DRR is accomplished. Finally we receive reconstructed 3D model in CAD system.

The database used in experiment contains 99 lumbar vertebras - 3D CAD models and DRR images Each vertebra has different geometry, and is described by FEM grid with the same structure (616 nodes, 2000 elements). For this database the Principal Component Analysis was done. The result of this operation is the mean object, 11 modes and coefficients. The first five modes include 96% of information about reconstructed geometry (Tab. 1). The twelfth and other modes contain very low information (this is only a numerical noise) and they aren't used for further reconstruction.

Modes describe the features of the vertebras (Fig. 5). The first and fourth mode describe the deformation of the vertebral body, the second, third and seventh mode represent the deformation of the spinosus process. Other modes describe the deformation of the transverse process.

Table 1. Participation of the modes in reconstruction.

Mode no.	Participation of the mode	Total partici-pation	Mode no.	Participation of the mode	Total partici-pation
1	60,7418270	60,7418270	7	0,6885226	97,9108118
2	18,7637347	79,5055617	8	0,5571059	98,4679177
3	8,0275130	87,5330747	9	0,5541704	99,0220881
4	7,5452177	95,0782924	10	0,5054029	99,5274910
5	1,1870943	96,2653867	11	0,4725086	99,9999996
6	0,9569025	97,2222892	12	0,0000002	99,9999998

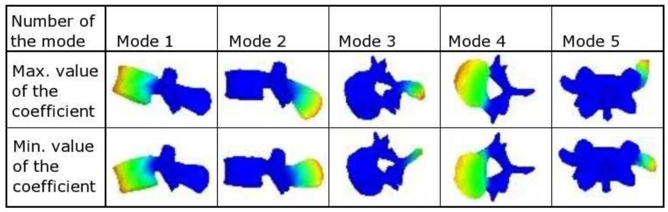

Figure 5. Three dimensional visualization of first 5 modes.

To verify this method the new vertebra has been made. It is deformed by three features (spinous process, vertebral body, transverse process) and has completely new values (there is no similar shape in data base).

In the next step we compare the DRR images of the searched and created vertebra and manipulate of the coefficients of modes. To compute the values of coefficients for all modes Jacobi criterion was used. The final result of this experiment is the solid CAD model (Fig. 6) and FEM grid.

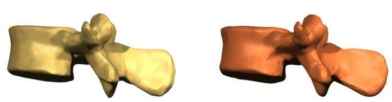

Figure 6. Solid model in CAD systems: searched and reconstructed vertebra.

To verify the quality of the reconstruction anthropometric measurements of the reconstructed vertebra was done. Mean inaccuracy of the reconstruction is about 0,25 mm (1%). Volume differences between searched and reconstructed vertebra is about 0,11% and surface difference is 0,95%.

5. Conclusions

Presented method makes possible a reconstruction of three-dimensional shape of the complex geometry in CAD system, basing on the few RTG images and knowledge about object geometry recorded in database.

This method uses full volume information from RTG images and they can be used for reconstruction of the biological and non-biological objects (e.g. mechanical objects).

Important property of developed algorithm is the possibility to change the geometry by manipulation of the coefficient values and creation of new 3D objects not existing in database.

Accuracy of the method is about 0,3 mm (comparable with 3D scanners). Important characteristic of the presented method is the automation of searching of the solution (elimination of landmarks – some methods require manually placed reference points for registration) and that this method is non-invasive and non-destructive. The viscous fluid registration makes possible reconstruction of the FEM grid with high precision and usage of the transfer area makes possible to get the correct topology of the grid nodes.

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