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DEVELOPMENT AND VALIDATION OF STRUCTURAL MODELS OF HUMAN POSTURE

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Abstract. A time law of small vibrations of the center of pressure (COP) of humans in standing position (human posture) provides a useful information about the physical and health condition of an individual. In this work controlled inverted pendulum (CIP) model with a single degree of freedom (DOF) is used in order to simulate the vibration of COP of a human body in anterio-posteriori (forward-backward) direction during still standing. A method for identification of the CIP model parameters is based on numerical computation of the sensitivities of the penalty-type error function to small variations of model parameters. The approach is applicable to structural models with any number of DOF and any structural complexity. Numerical example demonstrates the model parameter identification results providing nearly the best match of the model behaviour to the real experimental record of COP position.

Key words: controlled inverted pendulum, human posture, optimal control, sensitivity functions

1. Introduction

The time law of small vibrations of the center of pressure (COP) of humans in standing position (human posture) may provide useful information about the physical and health condition of an individual. One of the most popular ways to measure standing stability is to register movements of COP on the base of support. The resulting figure is called a stabilogram. COP signal represents a collective outcome of all systems that are responsible for maintaining body upright.

Different models are used in order to explain this, at the first glance, quite chaotic signal (see Fig. 1): pinned polymer [2], auto regressive [6] or fuzzy logic [4] models. One of the most popular is the controlled inverted pendulum (CIP) model which represents movements of COP in anterio-posteriori (forward-backward) direction during still standing. Traditionally the model
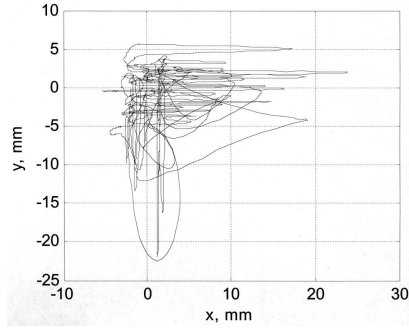
Figure 1. Movement of COP on a base of support.
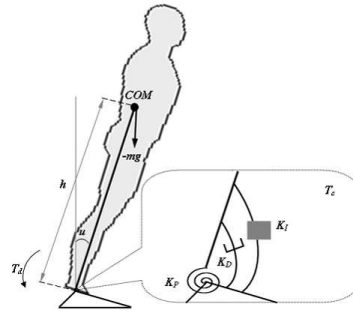
Figure 2. Controlled Inverted Pendulum (CIP) model of a human posture.

parameters are selected empirically by comparing the computed and experimental data [3, 5]. In this work a formal method for identification of the CIP model parameters developed by using the optimum control technique [1] is presented.

2. Methods

2.1. Controlled inverted pendulum model and error function formulation

In CIP model (Fig.2) human body is represented as a rigid body with the centre of mass (COM) oscillating around the ankle joint. The dynamic equation of such a system is obtained on the base of the angular momentum principle by making assumptions that the posture deviates away from upright position due to some physiological factors (e.g. breathing), as well as due to the torque generated by the gravity force. The influence of such factors in the CIP model is represented as the *cumulative disturbance torque* $T_d(t)$.

According to Peterka [5] the cumulative disturbance torque may be represented as low-filtered white noise. In order to counteract the disturbance torque human body produces the *corrective torque* $T_c(t)$. The corrective torque is assumed to be a linear function of the sway angle, angular velocity and the time integral of the sway angle. In [5] it has been referred to as *proportional, integral and differential (PID) controller*. Finally the single DOF CIP model is described as

$$I\ddot{u}(t) - mgh\,u(t) = T_d(t) - T_c(t), \qquad (2.1)$$

where: m is body mass; I mass moment of inertia of the body about the ankle joint; h distance of COM from the ankle joint; u sway angle; g gravitational acceleration; T_d *cumulative disturbance torque* obtained from the equation $T_d(t) + B\dot{T}_d(t) = Ax(t)$; $x(t)$ random number; A, B low-filter coefficients;

$$T_c(t) = K_P u(t) + K_I \int_t u(t)\,dt + K_D \dot{u}(t)$$

is corrective torque; K_P, K_I, K_D are coefficients.

Proper values of CIP model parameters K_P, K_I, K_D, A and B have to be determined in order to make the response of the model close to time law of vibrations of COP of a real human body. The quantitative measure of the deviation of the model behavior from the available experimental record is introduced by means of the chosen *error function* as

$$J(u) = \int_0^T \psi(u, \dot{u}, \ddot{u}) \ dt, \qquad (2.2)$$

where: $\psi(u, \dot{u}, \ddot{u}) = \frac{1}{2} (u_{COP}(t) - u_{ref}(t))^2$, u_{ref} are COP values recorded during experiment, u_{COP} are COP values calculated from CIP model data according to the formula presented in [5]: $u_{COP}(t) = hu(t) - \frac{I\ddot{u}(t)}{mg}$.

2.2. Sensitivity coefficients of the error function

2.2.1. Method

Consider an elastic structure presented by the dynamic equation as

$$m\ddot{u} + c\dot{u} + ku = w(u, \dot{u}, \{p\}) + rf,$$

where: u, $\dot{u}$, $\ddot{u}$ are displacement, velocity and acceleration of a body; w is non-linear force vector; f input excitation vector; $r=const$, which converts an input excitation vector to the nodal force vector; $\{p\}$ is a vector of model parameters.

The objective is to find such f and $\{p\}$ values in time interval $[0,T]$, which minimize the target function

$$J = \varphi(u_T, \dot{u}_T) + \int_0^T \psi(u, \dot{u}, \ddot{u}\{p\}) \ dt, \qquad (2.3)$$

where: $u, \dot{u}, \ddot{u}$ are displacement, velocity and acceleration at the end of interval $(t = T)$. The minimum value of the target function J is ensured when:

$$\frac{\partial J}{\partial f} = 0, \quad \frac{\partial J}{\partial p} = 0, \quad \frac{\partial J}{\partial T} = 0. \qquad (2.4)$$

However, in practice it often appears too difficult to solve the system of nonlinear equations (2.4). Therefore the iterative procedure is applied for minimization of this function, with sensitivity functions $\frac{\partial J}{\partial \{p\}}$, $\frac{\partial J}{\partial f}$, $\frac{\partial J}{\partial T}$ as the search direction.

The techniques for obtaining the sensitivity functions when applied to structural dynamics optimization problems are thoroughly discussed in [1]. Conjugate variables are used in order to express the variation of the target function in terms of $\{p\}$ and $f(t)$. The basic variation relation reads as:

$$\delta J = \frac{\partial J}{\partial T}\delta T + \frac{\partial J}{\partial p}\delta p + \int_0^T \frac{\partial J}{\partial f}\delta f \; dt, \tag{2.5}$$

where:

$$\frac{\partial J}{\partial f} = (\lambda + \dot{\mu} + \ddot{\eta})r, \quad \frac{\partial J}{\partial p} = \int_0^T (\lambda + \dot{\mu} + \ddot{\eta})\left(\frac{\partial w(u_T, \dot{u}_T, p)}{\partial p}\delta p + \frac{\partial \psi}{\partial \{p\}}\right) \; dt,$$

$$\frac{\partial J}{\partial T} = \psi(u_T, \dot{u}_T, \ddot{u}_T, \{p\}) - (\lambda_T + \dot{\mu}_T + \ddot{\eta}_T)(c\dot{u}_T + ku_T - w(u_T, \ddot{u}_T, p) - rf_T)$$
$$- \ddot{u}_T(\dot{\lambda}_T m - \dot{\eta}_T \tilde{c}_T + \eta_T \tilde{k}_T + \ddot{\eta}_T m) + \dot{u}_T(\lambda_T \tilde{c}_T + \mu_T \tilde{k}_T + \dot{\eta}_T \tilde{k}_T).$$

$\lambda(t)$, $\mu(t)$, $\eta(t)$ are conjugate variables the time laws of which are obtained integrating in time the differential equations

$$\begin{cases} \ddot{\lambda}m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \dot{\eta}\dot{\tilde{k}} = \frac{\partial \psi}{\partial u}, \\[2mm] \ddot{\mu}m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\tilde{k} = -\frac{\partial \psi}{\partial \dot{u}}, \\[2mm] \ddot{\eta}m - \dot{\eta}\tilde{c} + \eta\tilde{k} = \frac{\partial \psi}{\partial \ddot{u}} \end{cases} \tag{2.6}$$

with boundary conditions

$$\lambda_T = \dot{\lambda}_T = \dot{\mu}_T = \dot{\eta}_T = 0,$$

$$\mu_T = \frac{\partial \varphi}{\partial u_T}, \quad \eta_T = -\tilde{k}_T^{-1}\frac{\partial \varphi}{\partial \dot{u}_T}, \tag{2.7}$$

where: $\tilde{c} = c - \dfrac{\partial w(u, \dot{u}, p)}{\partial \dot{u}}$, $\tilde{k} = k - \dfrac{\partial w(u, \dot{u}, p)}{\partial u}$.

2.2.2. Application to CIP model parameter identification problem

Differential equation of CIP model (2.1) is considered together with the error function (2.2) used as a target function.

In our CIP model the control time T and excitation function f are assumed as fixed, so we obtain:

$$\partial T = 0, \;\; \partial f = 0, \;\; \frac{\partial J}{\partial T} = 0, \quad \frac{\partial J}{\partial f} = 0, \;\; \frac{\partial \psi}{\partial \dot{u}} = 0;$$

$$\frac{\partial \psi}{\partial u} = \frac{\partial}{\partial u}\left(\frac{1}{2}\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{ref}(t)\right)^2\right) = h\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{ref}(t)\right);$$

$$\frac{\partial \psi}{\partial \ddot{u}} = \frac{\partial}{\partial \ddot{u}}\left(\frac{1}{2}\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{ref}(t)\right)^2\right) = -\frac{I}{mg}\left(hu(t) - \frac{I\ddot{u}(t)}{mg} - u_{ref}(t)\right).$$

Equations (2.6) take the form:

$$\begin{cases} \ddot{\lambda}m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \dot{\eta}\dot{\tilde{k}} = h\big(hu(t) - \dfrac{I\ddot{u}(t)}{mg} - u_{ref}(t)\big), \\[2mm] \ddot{\mu}m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\dot{\tilde{k}} = 0, \\[2mm] \ddot{\eta}m - \dot{\eta}\tilde{c} + \eta\tilde{k} = -\dfrac{I}{mg}\big(hu(t) - \dfrac{I\ddot{u}(t)}{mg} - u_{ref}(t)\big) \end{cases}$$

with initial conditions derived from equations (2.3) and (2.7):

$$\lambda_T = \dot{\lambda}_T = \mu_T = \dot{\mu}_T = \eta_T = \dot{\eta}_T = 0.$$

The derivative $\frac{\partial J}{\partial p}$ is calculated from equation (2.5). Since $\frac{\partial \psi}{\partial \{p\}} = 0$, we obtain

$$\frac{\partial J}{\partial p} = \int_0^T \big((\lambda + \dot{\mu} + \ddot{\eta}) \frac{\partial w(u_T, \dot{u}_T, p)}{\partial p}\big) \, dt,$$

where:

$$w(u, \dot{u}, \{p\}) = Bx(t) - A\dot{y}(t) - \big(K_P u + K_I \int_0^I u \, dt + K_D \dot{u}\big).$$

Therefore sensitivity function $\frac{\partial J}{\partial p}$ reads as

$$\frac{\partial J}{\partial p} = \begin{cases} \int_0^T \big(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))u(t)\big) \, dt; \\[2mm] \int_0^T \big(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t)) \int_0^t u(\tau) d\tau\big) \, dt; \\[2mm] \int_0^T \big(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))\dot{u}(t)\big) dt; \\[2mm] \int_0^T \big(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))\dot{y}(t)\big) \, dt; \\[2mm] \int_0^T \big((\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))x(t)\big) \, dt \end{cases} \qquad (2.8)$$

In order to minimize the target function J and solve equation (2.8) the Steepest Descent method was used.

2.2.3. Implementation

The CIP model parameter identification algorithm was implemented in Matlab7. White noise time signal used to generate disturbance torque was produced by Matlab function "randn". As reference signal U_{ref} we used the COP signal recorded during experiments by using sample rate $10\,\text{Hz}$ during 60 s. Body mass and height of COM are measured data of an individual person. We used the mass moment of inertia $I{=}76$ kg· m^2, mass $m{=}60$ kg and distance of COM from the ankle $h = 1.13\,\text{m}$.

3. Results of CIP Model Parameter Identification

The experiments were conducted in order to find the best coincidence of the experimental COP signal (Figure 3, signal $Ucop_{ref}$) with the COP signal produced by CIP model (Figure 3, $Ucop_{before}$ – signal before parameters identification, Ucop – after). A set of experiments was conducted in order to identify parameters of corrective torque T_c, keeping the same signal of disturbance torque T_d. Initial corrective torque and disturbance torque parameters have been set the same as in the experiment described in [5] and commented as being able to produce a realistic COP signal. During experiments the values of K_P, K_I and K_D changed very slightly and the resulting signal *Ucop* did not changed significantly (Fig. 3).
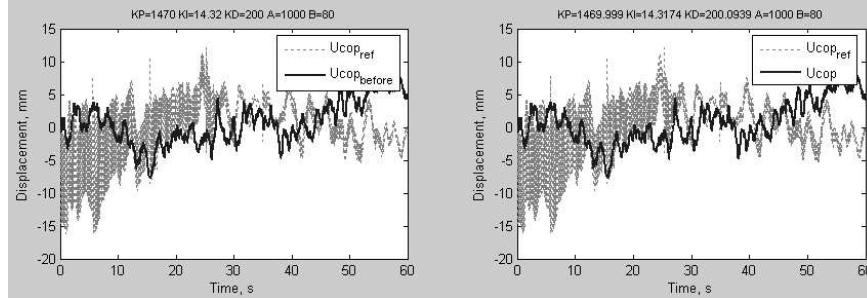
Figure 3. Parameter identification of corrective torque T_c.

The second set of experiments was conducted in order to investigate the influence of parameters of disturbance torque T_d on the CIP model performance. During this experiment the model parameters $K_P = 1470$, $K_I = 14.32$, $K_D = 200$ were kept constant, and identification of coefficients A and B performed by means of the error function minimization procedure. As a result, coefficient A decreased from 1000 to 937 while the filter coefficient B increased from 80 to 140. The absolute value of the error function decreased form $1.277\mathrm{e}^{-3}$ to $9.8356\mathrm{e}^{-4}$. In Figure 4 T_d signal represents CIP model disturbance torque before parameter identification and Td_{opt} – disturbance with improved values of parameters.

The last set of experiments was conducted in order to identify both excitation (disturbance) parameters A, B and model (control) parameters K_P, K_I and K_D. The initial values of model and excitation parameters were set the same as in *R.* J. Peterka's experiment[5] (upper left graph of Figure 5). In upper right graph of Figure 5 the identified CIP model and disturbance torque parameters are shown. Ucop represents the COP signal generated by CIP model with identified parameters: $K_P = 1469$, $K_I = 14.3$, $K_D = 198$, $A = 1101$, $B = 361$. The absolute value of the target function J decreased form $1.277\mathrm{e}^{-3}$ to $8.65\mathrm{e}^{-4}$ (lower left graph of Figure 5). Lower right graph of Fig-
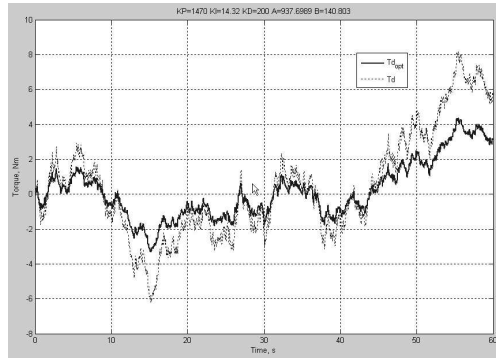
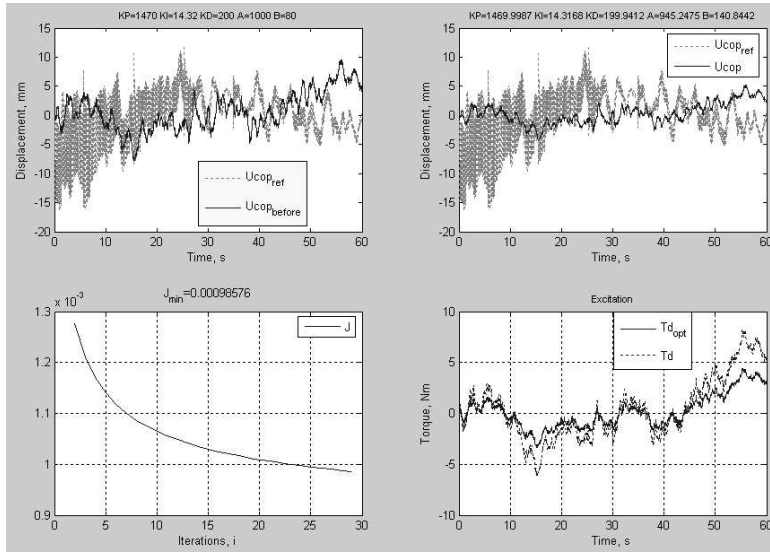**Figure 4.** Optimization of disturbance torque T_d parameters.

Figure 5. Model and excitation parameter identification.

ure 5 demonstrates how initial excitation T_d was transformed to model identified excitation Td_{opt}.

It can be visually inspected from the Figure 5 that the COP signal of the model (*Ucop* in top right graph of Figure 5) is highly dependant on disturbance torque T_d (Td_{opt} in bottom right graph).

The experiments revealed that CIP model due to its simplicity was not able to present very high and very low frequency components of COP signal simultaneously: in upper right graph of Figure 5 $Ucop_{ref}$ signal have both high and low frequency components while Ucop signal presents only a subset of real COP frequency components.

4. Conclusions

The single DOF controlled inverted pendulum human posture model was investigated in order to obtain parameter values which provide satisfactory coincidence between simulation and experimental results. General case of elastic structure optimal design and control method was investigated and extended to the case when the target function is dependant on the displacement, velocity and acceleration of the body. This method was used to identify CIP model parameters (disturbance and control). The error function presenting the cumulative non-coincidence of theoretical and experimentally recorded signal has been minimized with step-by-step procedure, where the gradient of the error function has been used as the search direction. As a result, optimum values of model parameters have been obtained.

The implemented CIP model presented the ability to repeat the recorded benchmark functions with acceptable tolerance. The experiments using real COP signal values revealed that CIP model due to its simplicity was not able to present higher and lower frequency components of COP signal simultaneously. Therefore more realistic structural models which are able to take into account both ankle and hip strategy of human posture should be investigated.

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