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NUMERICAL ANALYSIS OF THE HEAT AND MAGNETOHYDRODYNAMIC FLOWS IN A FINITE CYLINDER INDUCED BY HIGH FREQUENCY ELECTRIC CURRENT

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Abstract. The distribution of electromagnetic fields, forces and source term of temperature induced by an alternating axially-symmetric system of electric current in a cylinder of a finite length has been investigated and calculated in [1, 2, 3]. In this work the three-phase alternating current with high frequency ($f > 50Hz$) is fed to each of N discrete circular conductors-electrodes, which are placed on the internal wall of the cylinder. The magneto-hydrodynamic flow of viscous incompressible weakly electroconductive liquid-electrolyte and temperature are obtained by the finite difference method, using monotonous finite-difference schemes.

The average axially-symmetric motion of electrolyte and temperature distribution in a cylinder has been obtained in dependence of the values of frequency and arrangement of electrodes.
Key words: Magneto-hydrodynamic flow, temperature, monotonous finite-difference schemes, alternating current, high frequency

1. Introduction

In many technological applications it is important to mix and heat an electroconductive liquid, using various magnetic fields. One of the modern areas of applications developed during last years is effective use of electrical energy produced by alternating current in production of heat energy.

In traditional heating systems for the dwelling houses fuel is used to warm up the water, which flows threw the heating system. The offered mathematical model describes the function of such heating devices in which the water of the heating system is warmed up with the help of alternating current in one mode. Conclusion is that it helps to increase the efficiency of the device (there is no excess loss of heat) and that the device is extremely compact.

Devices based on this principle are developed during last ten years. This work presents the mathematical model of one of such devices. It is a finite cylinder with N metal coils-electrodes positioned on its inner surface with a fixed distance from each other. By connecting those coils to three–phase alternating current with height frequency, they irradiate energy.

In this work we consider a finite cylinder

$$\tilde{\Omega} = \{(r,z):\ 0 < r < a, 0 < z < Z\}$$

with N metal coils-electrodes

$$L_i = \{(r,z), r = a, z = z_i\}, \quad 0 < z_i < Z,\ i = \overline{1,N},$$

positioned on its inner surface with a fixed distance from each other. Alternating current with density $j_i = j_0 cos(\tilde{\omega}t + (i-1)\theta)$, is fed to each of N discrete circular conductors. Here $\tilde{\omega} = 2\pi f$, f and j_0 are the frequency and amplitude of the alternating current, $\theta = const$ is the phase (usually $\theta = 120^0$) and t is the time.

The current creates in the weakly conductive liquid-electrolyte the radial and axial components of the magnetic field as well as azimuthal component of the induced electric field which, in its turn, creates axial F_z and radial F_r components of the electromagnetic force (Lorentz' force) and heat sources term j_ϕ^2 (j_ϕ is the azimuthal component of the vector of induced current density).

For calculating the electromagnetic fields, the averaging method over the time interval $2\pi/\tilde{\omega} = 1/f$ is used. The averaged values of force $< F_r >$, $< F_z >$ give rise to a liquid (electrolyte) motion, which can be described by the stationary Navier-Stokes equation. The averaged value of source $< j_\phi^2 >$ give the distributions of the temperature in the cylinder.

At the inlet of the cylinder we have a uniform velocity $U_0 \approx 0.1\frac{m}{s}$. The liquid have following parameters: kinematic viscosity $\nu \approx 10^{-5}\frac{m^2}{s}$, density $\rho \approx 1000\frac{kg}{m^3}$, the electric conductivity $\sigma \approx 100\Omega^{-1}m^{-1}$, the specific heat capacity $c \approx 4000\frac{J}{kg.K}$, the heat conductivity $\lambda \approx 0.6\frac{W}{m.K}$ and the heat exchange coefficient $\alpha \approx 12\frac{w}{m^2.K}$. The radius a of the cylinder is $0.05m$, the length Z of the cylinder is $0.25m$.

The main aim of this work is to analyze some connection schemes of 9 electrodes and value of frequency influence of vortex and temperature formation in the cylinder.

2. The Navier-Stokes Equations and Heat Transfer Equation

The axially-symmetric stationary Navier-Stokes equations for vorticity function ω, and hydrodynamic-stream function ψ in the cylindrical coordinates (r, ϕ, z) are written in the following non-dimensional form ([2]- without circulation):

$$\begin{cases} \dfrac{Re}{r} J(\psi, \omega) = \dfrac{\partial^2 \omega}{\partial z^2} + \dfrac{1}{r^3} \dfrac{\partial}{\partial r}\Big(r^3 \dfrac{\partial \omega}{\partial r}\Big) + \dfrac{Re.Te. < f^{\phi} >}{r}, \\ \Delta^*(\psi) = -r^2 \omega, \end{cases} \tag{2.1}$$

where $J(\psi, \omega)$ is the Jacobian of the functions ψ and ω, $< f^{\phi} >$ is the averaged azimuthal component of the force curl vector's,

$$\Delta^*(\psi) = r\partial(r^{-1}\partial\psi/\partial r)/\partial r + \partial^2\psi/\partial z^2$$

is the conjugate expression for the Laplace operator, $\omega = r^{-1}\omega_{\phi}$, ω_{ϕ} is the azimuthal component of the velocity curl vector's, $v_r = -r^{-1}\partial\psi/\partial z$, $v_z = r^{-1}\partial\psi/\partial r$, are radial and axial components of the velocity, $Re = U_0 r_0/\nu$ is the Reynolds number, $Te = \sigma\tilde{\omega}(\mu j_0/2\pi U_0)^2/\rho$ is the Taylor number, $\mu = 4\pi\ 10^{-7}\ \frac{m.kg}{s^2.A^2}$ is the magnetic permeability in vacuum.

The equations (2.1) were put into the dimensionless form by scaling all the lengths to $r_0 = a$ (the inlet radius of the tube), the axial velocity v_z to U_0, the vorticity ω to $\omega_0 = U_0/r_0^2$ and stream function ψ to $\psi_0 = U_0 r_0^2$.

We get the following dimensionless form of the boundary conditions:

1) In the part of the inlet ($z = 0, 0 \leq r < r_1$) the axial streams with a uniform velocity U_0 give $\omega = 0, \quad \psi = 0.5r^2$; in the other part of the inlet ($z = 0, r_1 \leq r \leq 1$) we have $\omega = 0, \psi = 0.5(r_1^2 + \beta(r^2 - r_1^2))$, where $\beta \approx 0.1$ is the velocity ratio of the coaxial free stream velocity to axial jet velocity U_0.

2) The symmetry conditions along the axis ($r = 0$) is given by $\psi = \partial\psi/\partial r = \partial\omega/\partial r = 0$.

3) The outflow boundary conditions at the outlet ($z = l = Z/a$) are given by $\partial\psi/\partial z = \partial\omega/\partial z = 0$.

4) The boundary conditions at walls ($r = 1$) are given by

$$\psi_w = \frac{1}{2}(r_1^2 + \beta(1 - r_1^2)), \quad \omega = \omega_w,$$

where ω_w is the dimensionless wall-vorticity obtained within the frame of finite-difference method from no-slip conditions [2], r, r_1 are the dimensionless coordinates.

The axially-symmetric stationary distribution of temperature field is described by the following non-dimensional boundary-value problem for the heat transport equation [1, 2]:

$$\begin{cases} \dfrac{Pe}{r} J(\psi, T) = \Delta T + Ec\ Pr\ \Phi + K_T\ < j_{\phi}^2 >, \\ \dfrac{\partial T}{\partial r}\Big|_{r=1} = -Bi\ T|_{r=1}, T|_{z=0} = 0, \quad \dfrac{\partial T}{\partial z}\Big|_{z=l} = \dfrac{\partial T}{\partial r}\Big|_{r=0} = 0, \end{cases} \tag{2.2}$$

where $K_T = \dfrac{(\mu j_0 \tilde{\omega} a)^2 \sigma}{(2\pi)^2 \lambda (T_b - T_a)}$ is the heat sources parameter, T_a is the given constant external temperature (T_b is the limit temperature), $Bi = \alpha a/\lambda \approx 1$ is the Biot number, Φ is the dimensionless axially-symmetric dissipation function, $Pr = c\rho\nu/\lambda$, $Pe = PrRe$, $Ec = U_0^2/(cT_a)$ are the Prandtl, Peclet and Eckert numbers, $T = \dfrac{\bar{T} - T_a}{T_b - T_a}$ is the dimensionless temperature ($\bar{T}$ is the dimensional temperature).

3. The Finite-Difference Approximations and Numerical Results

The presence of large parameters at the first order derivatives (Re, Pe) in the systems of differential equations causes additional numerical difficulties for the application of general finite–difference methods. Thus special monotonous approximations are constructed [3, 4] by using the exponential functions

$$s(x) = \frac{x}{exp(x) - 1} > 0, \;\; s'(x) = \frac{ds}{dx} < 0, \;\; s(0) = 1, \;\; s'(0) = -\frac{1}{2}$$

with the Patankar approximations in the following form [5]:

$$s(x) = \max\left((1 - 0.1|x|)^5, 0\right) + \max(-x, 0).$$

We consider an uniform grid with the dimensionless steps 0.1.

As the basis for calculations 9 circular conductors L_i are chosen, which are arranged in the axial direction at the points

$$zj = [z_1, \; z_2, \; z_3, \; z_4, \; z_5, \; z_6, \; z_7, \; z_8, \; z_9],$$

where $z_i = 0.2i, i = \overline{1, 9}$. The results of numerical experiments for

$$< F_r >, \; < F_z >, \; < f^{\phi} >, \; \psi, \; \omega, \; T$$

were obtained in the case of

$$j_0 = 2.8210^4 \frac{A}{m^2}, \;\; f = 50, 250, 500; \;\; Re = 500, \; Te = 0.002f,$$
$$K_T = 80.010^{-7} f^2, \;\; Pr = 67, \;\; Ec = 10^{-8}, \;\; l = 2.5, \;\; T_b - T_a = 65, \;\; r_1 = 0.5.$$

The numerical results depend on the arrangement of electrodes and on the value of frequency. The values of averaged forces $< F_z >, < F_r >$, curl of forces $< f^{\phi} >$ and the maximal value of the source function $< j_{\phi}^2 >$ depending of the two arrangement of 9 conductors by numbering $nj = [123456789]$ are presented in the Table 1. This arrangement is used in [2] and it gives the maximal temperature in the cylinder.

In Figures 1-4 we can see the vortex and temperature formation in the cylinder depending on the parameter of frequency f.

Table 1. The extremal values of averaged forces and curl of forces.

No	nj	$< F_z >$	$< F_r >$	$< f^{\phi} >$	$< j_{\phi}^2 >$
1	[147258369]	[-69.0;3.50]	[-36.9;36.9]	[-0.2;188]	12.9
2	[963852741]	[-3.50;69.0]	[-50.5;50.5]	[-200;49.]	12.9

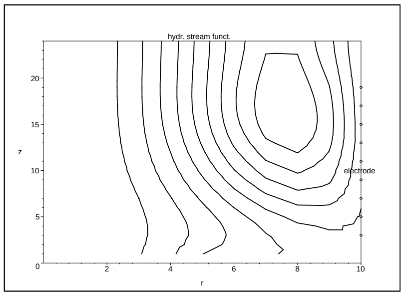

Figure 1. The stream functions $\psi \in (0.00, 0.34), f = 50$.

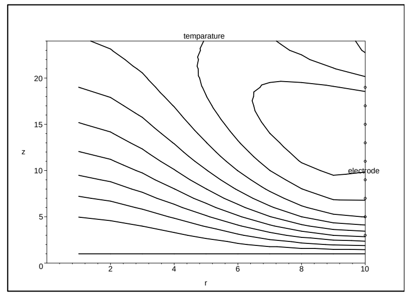

Figure 2. The temperature $T_{max} = 0.02, T_{av} = 0.007, f = 50$.

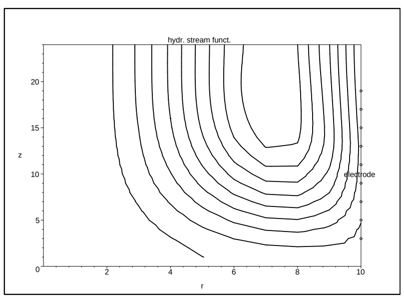

Figure 3. The stream functions $\psi \in (0.00, 1.02), f = 500$.

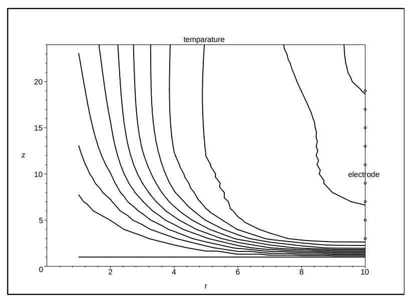

Figure 4. The temperature $T_{max} = 0.80, T_{av} = 0.33, f = 500$.

We obtain the following results for the dimensionless values of ψ_{max}, $\omega \in [\omega_{min}, \omega_{max}]$, T_{max}, $T_{av} = l^{-1} \int_0^l \int_0^1 rT(r,z)drdz$ (the average dimensionless temperature) depending on two connections of electrodes and on the parameters f, Te, K_T:

1. Conductors are connected to each other skipping two of them, the ends of 3 wires are in the beginning of electrodes nj = [147258369](see Table 1, case No = 1):

$$a) f = 50, \;\; Te = 0.1, \;\; K_T = 0.02, \;\; \psi_{max} = 0.34,$$
$$\omega \in [-7, 13], \;\; T_{max} = 0.02, \;\; T_{av} = 0.007.$$

Fig. 1 shows large vortex induced by the Lorentz force at the last electrode, the orientation of the vortex is clockwise.

Fig. 2 shows the distribution of temperature:

$$b) f = 250, \;\; Te = 0.5, \;\; K_T = 0.5, \;\; \psi_{max} = 0.72,$$
$$\omega \in [-22, 35], \;\; T_{max} = 0.26, \;\; T_{av} = 0.10,$$

the vortex by the electrodes and the maximal value of temperature increases in the cylinder.

$$c) f = 500, \;\; Te = 1.0, \;\; K_T = 2.0, \;\; \psi_{max} = 1.02,$$
$$\omega \in [-35, 52], \;\; T_{max} = 0.80, \;\; T_{av} = 0.33.$$

Fig. 3 shows that this vortex increases, Fig. 4 shows the distribution of the temperature.

2. The ends of 3 wires are in the end of electrodes, $nj = [963852741]$ (see Table 1, case No = 2):

$$a) f = 50, \;\; Te = 0.1, \;\; K_T = 0.02, \;\; \psi \in [-0.23, 0.16],$$
$$\omega \in [-17, 10], \;\; T_{max} = 0.01, \;\; T_{av} = 0.004,$$

we have large vortex by the first electrodes in opposite direction, induced by the Lorentz force;

$$b) f = 250, \;\; Te = K_T = 0.5, \;\; \psi \in [-0.75, 0.16],$$
$$\omega \in [-40, 9], \;\; T_{max} = 0.16, \;\; T_{av} = 0.07,$$

the vortex by the electrodes increases,

$$c) f = 500, \;\; Te = 1.0, \;\; K_T = 2.0, \;\; \psi \in [-1.28, 0.15],$$
$$\omega \in [-61, 11], \;\; T_{max} = 0.60, \;\; T_{av} = 0.27,$$

this vortex increases.

4. Conclusion

1. The results of the numerical experiments with 9 circular conductors reported here can give some new physical conclusions on the flow behavior in the cylinder.

2. The averaged values of the electric field, electromagnetic forces and the azimuthal component of the curl of forces' vector are calculated for two arrangement of the electrodes and for different value of frequency.
3. The results of calculations show that the increase of the frequency enlarges the maximum temperature and the total produced heat.
4. From two different schemes of connection of electrodes, more efficient is the one where the electrodes are joined skipping two of the electrodes.

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