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# MATHEMATICAL MODELLING OF THE HEAT TRANSFER IN A MULTI-WIRE BUNDLE

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**Abstract.** A reduced order mathematical model is proposed for analysis of the temperature distribution in 2D multi-wire bundle. This model is compared with the full 2D mathematical model. For the evaluation of the accuracy of the smaller model the heat exchange rate in the bundle was determined and the reduced model was fitted to experimental data by varying the convection coefficient. The results of computational experiments are presented and some preliminary conclusions are done.

**Key words:** mathematical models, reduced models, multi-wire bundle, heat conduction

## 1. Introduction

In this paper we compare two models for calculation of temperature distribution in 2D multi-wire bundle.

The first model is most general. It describes in detail the multi-wire bundle consisting of 42 single cored insulated wires. The cross sections of wires are varying from 0.35 till 4 $mm^2$ and positions of cables are taken into account. The outside boundary condition considers the heat convection and radiation at the outer side of the bundle insulation. This problem is solved by the finite element method software package *ANSYS*.

In order to optimize the multi-wire bundles used in car industry we need to compute many times the solution of the direct problem with different values of parameters. Thus it is necessary to reduce large model to much smaller one.

This reduction must be such that the dominant behaviuor of solutions is similar for the small and large systems [1]. The model reduction step requires the of sophisticated numerical techniques, and is the subject of intensive research.

A very simple model was obtained in [2]. It is based on application of the homogenization procedure and 1D model of the quasi-homogeneous cable is derived. We note that this derivation should be considered as engineering heuristic and no strict mathematical analysis is presented in [2] to estimate the accuracy of the obtained 1D model. The results of computational experiments have proved that in many cases the model gives sufficiently accurate results.

In this paper we propose a reduced order intermediate mathematical model. It is based on the following assumptions:

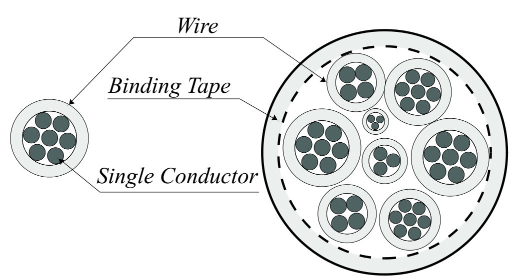
- Each single cored insulated wire is described by using a radial symmetry assumption;
- The heat convection and radiation takes place at the outer side of each wire isolation. Here we use our main assumption, that the "virtual" temperature of air is constant at each time moment inside the bundle and it can be calculated from the heat conservation equation.

The rest of the paper is organized as follows. In Section 2, we formulate non–stationary 2D mathematical model. It solved by using *ANSYS* software package. In Section 3, we present the reduced mathematical model and describe the finite-volume scheme for solving the given initial–boundary value problems. In Section 5, we discuss numerical experiments with both models.

## 2. Two Dimensional Mathematical Model

The multi-wire cable consists of $M$ various single insulated electric wires, see Fig. 1.

a)




b)

**Figure 1.** The multi–wire cable: a) photo image, b) diagram.

The mathematical model of multi-wire cable consists of non-stationary nonlinear parabolic PDE for anisotropic media, since the wires are made of coper conductor and PVC insulation. The equation which describes the distribution of the temperature can be written as:

$$c(T)\rho\frac{\partial T}{\partial t} = \sum_{i=1}^{2}\frac{\partial}{\partial x_i}\Big(k(X)\frac{\partial T}{\partial x_i}\Big) + f(T,X), \quad (X,t)\in D\times(0,t_F], \tag{2.1}$$

where $D$ is the region defining a cross section of bundle, $k$ is heat conductivity coefficient, $c$ is specific heat capacity, $\rho$ is density, $f$ is volumetric heat generation source.

We suppose, that temperature $T(X,t)$ is continuous function in $D$, and the change of heat flux at boundaries of materials with different properties is equal to zero. Boundary conditions on the exterior part of insulation surface are given by

$$k(X)\frac{\partial T}{\partial n} = \alpha_C(d,T)(T(X,t)-T_{env}) + \varepsilon\sigma\big(T^4(X,t)-T_{env}^4\big), \quad X\in\partial D, \tag{2.2}$$

where $\varepsilon$ is emissivity coefficient, $\sigma$ is the Stefan–Boltzmann constant, $T_{env}$ is absolute environment temperature, $d$ is a diameter of multiwire bundle. At the initial time moment the temperature of wires coincides with the environment temperature

$$T(X,0) = T_{env}, \quad X\in D\,. \tag{2.3}$$

Problem (2.1)–(2.3) is solved by commercial finite element software package *ANSYS*, which is widely used in computer-aided engineering technology and engineering design analysis.

## 3. The Reduced Mathematical Model

Each single insulated wire is described by using a radial symmetry assumption, thus we obtain a system of 1D heat conduction equations:

$$c(T_j)\rho\frac{\partial T_j}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\Big(rk(r)\frac{\partial T_j}{\partial r}\Big) + f(T_j,r), \quad (r,t)\in D_j\times(0,t_F], \tag{3.1}$$

where $D_j$ is the region of $j$-th wire. A similar equation is also written for the exterior insulation layer. Thus we get $(M+1)$ equations, which can be solved in parallel by using data parallel algorithms. Semi-automatic parallelization of this part of the algorithm can be done by using *ParSol* tool [3].

All these equations are nonlinearly connected due to boundary conditions:

$$-k(r)\frac{\partial T_j}{\partial r}\Big|_{r=R_j} = \alpha_C(R_j,T_j)\big(T(R_j,t)-T_a\big) + \varepsilon\sigma\big(T^4(R_j,t)-T_a^4\big), \tag{3.2}$$

where $T_a$ is a virtual air temperature. It is assumed that $T_a$ is constant in the whole air region $D_a$ (let us remind, that air fills up void places of the bundle).

Each PDE is approximated by standard implicit finite–volume scheme. Time derivatives are approximated by backward Euler scheme. The Newton method is used to solve obtained systems of nonlinear equations.

Next we will get additional equation which defines $T_a$. We start from a general equation for air temperature in $D_a$:

$$c_a(T)\rho_a \frac{\partial T}{\partial t} = \sum_{i=1}^{2} \frac{\partial}{\partial x_i}\Big(k_a(X)\frac{\partial T}{\partial x_i}\Big), \quad (X,t) \in D_a \times (0, t_F]. \tag{3.3}$$

Integrating equation (3.3) in $D_a$ and using assumptions that fluxes are continuous functions and $T(X,t^n) \approx T_a^n$, we get the following explicit equation

$$T_a^{n+1} = T_a^n + \frac{\tau \sum\limits_{j=0}^{M} \Big(\alpha_C(R_j,T_j)\big(T(R_j,t^n) - T_a^n\big) + \varepsilon\sigma\big(T^4(R_j,t^n) - T_a^{n4}\big)\Big)}{c_a(T_a^n)\rho_a}.$$

The complexity of one iteration is $\mathcal{O}(MN)$, where $N$ is the number of space grid points used to approximate $D_j$. Thus the proposed mathematical model of the reduced order can be solved very efficiently.

## 4. Discussion

First we have attempted to validate the reduced mathematical model. The convection coefficient $\alpha_c$ was fitted to make the results obtained by using the simplified model to be close to results obtained from the full 2D mathematical model. Experimental results show that the distribution of temperature in multiwire bundles of the reduced model is in acceptable agreement with those, which were obtained by simulation with *ANSYS* package. The error in all cases does not exceed a ten percent limit.

Our final aim was to validate our reduced order model for various bundles, in which wires are distributed randomly and have different cross sections. A number of simple bundle structures were investigated (see Table 1), where values of the temperature at the center of a bundle are presented.

**Table 1.** Experimental results for different examples of bundles.

| No. of wires $M$ | Load, in $W/m^3$ | 2D model $T(0,t_f)$ | Reduced order model | Error, in % |
|---|---|---|---|---|
| 3 | 1.6E6 | 111 | 121 | 8.3 |
| 7 | 1.6E6 | 163 | 170 | 4.1 |
| 19 | 5.9E5 | 136 | 140 | 2.9 |
| 37 | 5.9E5 | 187 | 188 | 0.5 |
| 42 | 4.7E5 | 95 | 100 | 5.0 |
| 61 | 2.8E5 | 150 | 146 | 2.7 |
| 91 | 2.8E5 | 182 | 183 | 0.5 |

By fitting the convection coefficient a good agreement of both 2D *ANSYS* and the reduced order models was received. Having these convection coefficients we can calculate the temperature with a sufficient precision for different bundles with randomly distributed wires.

## 5. Conclusions

We can make the following principle conclusions from the results of mathematical modelling:

1. The proposed mathematical model of reduced order enables us to compute an accurate approximation of the distribution of temperature in a multi-wire bundle.
2. The fitted convection coefficient can be represented as a function of the wire number in a bundle. Thus various configurations of bundles can be simulated later with such model.
3. It is important to generalize this model for description of steady state situation, since in most cases only the steady state is considered for optimization of multi-wire bundles.

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