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SIMULATION OF MULTI-PHASE CONTAMINANT TRANSPORT IN POROUS MEDIA USING PARALLEL COMPUTER SYSTEMS[1]

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Abstract. The research deals with parallel simulation of multi-phase immiscible incompressible fluid flows in porous media where one of fluids is a contaminant or contains a contaminant. Two types of models are used in computations – without and with capillary as well as gravity forces. Numerical implementations are based mostly on parallel algorithms of the implicit type.

Key words: porous media flow, oil recovery, infiltration, parallel implicit algorithm

1. Introduction

Simulation of fluid flows in porous media is of great practical importance. While developing oil recovery technologies and investigating ecology problems it is necessary to predict these flows. Usage of parallel computers allows us to solve such large-scale problems in an acceptable time.

The number of phases in porous media flows does not produce an essential influence on the character of governing models. Therefore it is enough to discuss models of two-phase flows. In this paper the classical Buckley-Leverett model neglecting capillary and gravity forces as well as the model taking into account capillary and gravity forces is considered. As a typical example for the first case the problem of passive contaminant transport in an oil-bearing stratum at water flooding is treated [2]. The second problem studied

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here is concerned with an oil spot on the earth surface infiltrating into the ground under the influence of the gravity force. The problem is similar in some sense to the DNAPL infiltration test problem from [1]. The primary goal is to investigate time-evolution of the contamination domain.

This research addresses mainly to parallel implicit algorithms. Some algorithms have been successfully implemented by the authors early in works [3], [4]. A natural extension of them is presented in this paper.

2. Contaminant Transport in an Oil-Bearing Stratum

Let us consider a coupled flow of oil and water in an oil field where oil is extracted by means of non-piston water displacement [2]. The field is covered by a network of vertical water injection and oil production wells (sources and sinks). The flow of two phases w (water) and o (oil) is governed by the classical Buckley-Leverett model [2]. Fluids are immiscible and incompressible, the medium is undeformable, phase flows satisfy the Darcy law, capillary and gravity forces are neglected. The following transformed system of equations is used:

$$m\frac{\partial s_w}{\partial t} + \mathrm{div}\Big(F(s_w)K(s_w)\,\mathbf{grad}P\Big) = \begin{cases} qF(\bar{s}) & \text{at sources,} \\ qF(s_w) & \text{beyond sources,} \end{cases} \tag{2.1}$$

$$\mathrm{div}\Big(K(s_w)\,\mathbf{grad}P\Big) = q. \tag{2.2}$$

The water saturation s_w and the pressure P are sought in a subdomain of symmetry cut from an unbounded uniform stratum. Here m is the porosity, q describes debits of wells, $F(s_w)$ is the Buckley-Leverett function, $K(s_w)$ is a nonlinear coefficient including the absolute permeability k, relative phase permeabilities $k_w(s_w)$, $k_o(s_w)$, dynamic viscosities μ_w, μ_o.

Rectangular grids of the MAC type are used for discretisation. Solution of transport equation (2.1) faces significant difficulties due to discontinuity in the saturation. Different explicit and implicit methods for solving this equation were studied [3]. In the current work it is solved by the iterative secant method and we demonstrate some its advantages over explicit schemes. Equation (2.2) is solved via the local relaxation (ad hoc SOR) with red-black data ordering [4].

In order to account the contaminant transport, the above model should be extended. Let us suppose that a passive contaminant arrives into the stratum with water (for example, salty water is injected at sources). The Darcy velocity of water $\mathbf{W}_w$ is obtained via the generalized Darcy Law:

$$\mathbf{W}_w = -k\frac{k_w(s_w)}{\mu_w}\,\mathbf{grad}P. \tag{2.3}$$

The equation for the contaminant concentration can be written like this:

$$m\frac{\partial(s_w c + a)}{\partial t} + \mathrm{div}\Big(\mathbf{W}_w c + \mathbf{S}(\mathbf{W}_w)\Big) = Q_c. \tag{2.4}$$

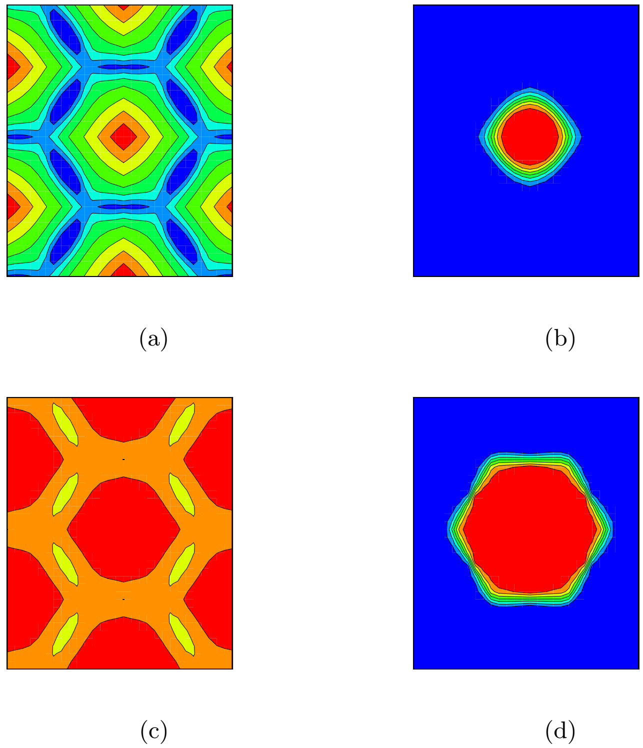

Figure 1. Water saturation (a, c) and contaminant concentration (b, d) fields at different time moments: (a, b) – after 20 days, (c, d) – after 200 days.

Here c is the contaminant solution concentration, a is the contaminant being adsorbed. Q_c describes sources of the contaminant. $\mathbf{S}$ is a "diffusion flow" caused by the convective diffusion:

$$S_i = -D_{ij} \frac{\partial c}{\partial x_j}, \quad i, j \in \{x, y\}.$$

D_{ij} is an effective tensor of the convective diffusion. There is the phenomenological formula by Nikolaevskij to obtain it:

$$D_{ij} = [(\lambda_1 - \lambda_2)\delta_{ij} + \lambda_2 n_i n_j] W_w,$$

where $W_w = |\mathbf{W}_w|$, n_i is the unit vector in the direction of $\mathbf{W}_w$, λ_1 and λ_2 are positive coefficients, their order of magnitude agrees with the character size of the medium microheterogeneity.

In this paper adsorption is not taken into account. Equation (2.4) is approximated by central differences. As (2.4) contains mixed derivatives the nine-point stencil is used. The computational domain is closed to a tore in both directions in order to provide periodic boundary conditions. The twolevel scheme with the weight 1/2 is used to provide the second order of approximation both in time and in space. The obtained difference equation is solved by the local relaxation method with special data ordering, when all nodes are subdivided into four groups.

The so-called honeycomb well configuration was considered. At the initial time moment $s_w = 0.1$, $P = 100\ atm$, $c = 0$ are specified over the whole domain. A contaminant enters the stratum with water through the central well. Fig. 1 presents saturation and concentration fields at different time moments.

3. Oil Infiltration into the Ground

The next problem deals with an oil spot on the earth surface infiltrating into the ground under the influence of the gravity force. Oil is treated as a contaminant. The problem is considered in the 2D vertical section in a fully air-saturated reservoir (see Fig. 2).

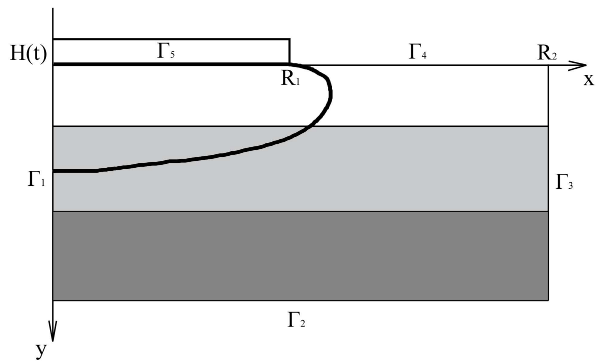

Figure 2. Oil spot infiltration into the ground.

$$\mathbf{W}_i = -k\frac{k_i(s_i)}{\mu_i}(\mathbf{grad}P_i - \rho_i g\mathbf{e}), \quad i = 1,2, \quad \mathbf{e} = (0,1) \tag{3.1}$$

$$m\frac{\partial s_i}{\partial t} + \mathrm{div}(\mathbf{W}_i) = 0, \quad i = 1,2 \tag{3.2}$$

$$s_1 + s_2 = 1 \quad \text{(index 1 corresponds to oil, index 2 − to air)} \tag{3.3}$$

$$P_1 - P_2 = P_k(s) = \sigma\cos\theta\sqrt{\frac{m}{k}}J(s), \quad s = s_1 - \text{here and below} \tag{3.4}$$

The following formula is used for the Leverett function:

$$J(s) = \begin{cases} J_0 \times (\bar{s}/\underline{s} - 1), & 0 \leq s \leq \underline{s} \\ J_0 \times (\bar{s}/s - 1), & \underline{s} < s \leq \bar{s} \\ 0, & \bar{s} < s \leq 1 \end{cases} \quad J_0 = 0.1,\ \underline{s} = 0.1,\ \bar{s} = 0.9 \tag{3.5}$$

Initial conditions are given by:

$$s^{(0)} = \underline{s},\ \ P_2^{(0)} = P_{atm} + \rho_2 g y.$$

Boundary conditions (in notations of Fig. 2): Γ_1 – the principal plane; Γ_2 – the impenetrable bed; Γ_3 – the permeable bed; Γ_4 – the earth surface; Γ_5 – the spot surface. At Γ_5 conditions are changed: 1) $P_2 = P_{atm} + \rho_1 g H(t),\ s = \bar{s}$ – till the spot exists; 2) the same conditions as at Γ_4 – when $H(t) \leq 0$. The oil spot has a fixed volume and can spread from the surface into the underground completely.

For computations the system (3.1) – (3.4) is reduced to two equations for the oil saturation and the air pressure. They are solved by IMPES method, the local relaxation method is used to obtain P_2. The goal of analysis is to investigate the time-evolution of the contamination domain. Fig. 3 presents results obtained for a homogeneous medium and for the heterogeneous three-layer medium. Permeability varies by two orders of magnitude.

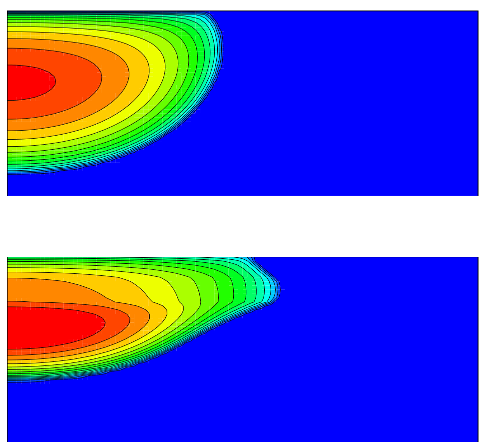

Figure 3. Oil saturation at the final moment for homogeneous (upper) and heterogeneous (lower) media.

The next considered problem is similar to the vertical infiltration of a DNAPL (dense nonaqueous phase liquid) into a fully water-saturated reservoir [1] but employs a different Leverett function and oil and water as components. A low permeable lens is placed into the reservoir. The capillary pressure is given by (3.4), (3.5). Two cases are investigated: permeability of the lens and of the rest of the domain varies by one order of magnitude or by five orders of magnitude. Fig. 4 demonstrates the obtained oil spreading.

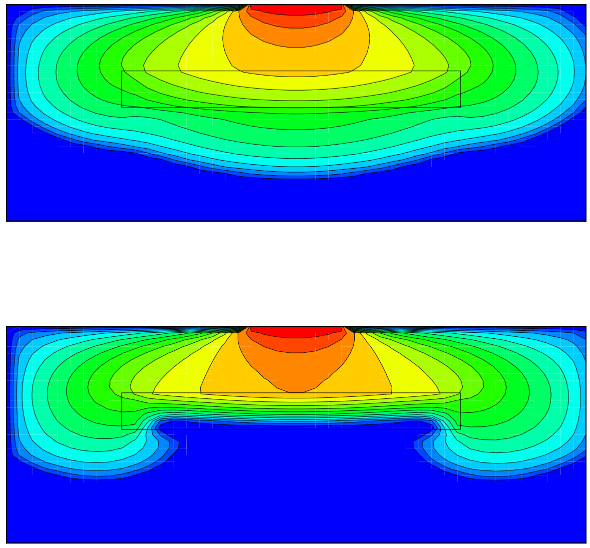

Figure 4. Oil saturation at the overflow moment for high and low permeable lenses.

4. Parallel Implementation

Parallel algorithms on the basis of computational domain partitioning were developed for solving system (2.1)-(2.2) in the oil recovery problem [3, 4], and for solving the oil infiltration problem as a whole. Moderate parallelization efficiency was achieved. For example, efficiency of the Fortran/PARIX code simulating oil recovery based on the fully implicit algorithm was about 0.65 on 4 processors of Parsytec PowerXplorer (the computational grid size was 117×143). Efficiency of the C++/MPI code for simulation of oil infiltration implementing IMPES method on 5 processors of our cluster (Intel Pentium III with Fast Ethernet) was about 0.73 (the grid size was 90×65).

5. Conclusion: Future Trends of Research

New accurate and efficient parallel algorithms will be studied and incorporated into the parallel software developed earlier for porous media flow simulation. In contrast to IMPES strategy, a fully implicit algorithm for solving the infiltration problem will be developed in order to increase the time step and to reduce the total run time. It is planned to simulate contaminant spreading in heterogeneous media with randomly distributed anisotropic properties.

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