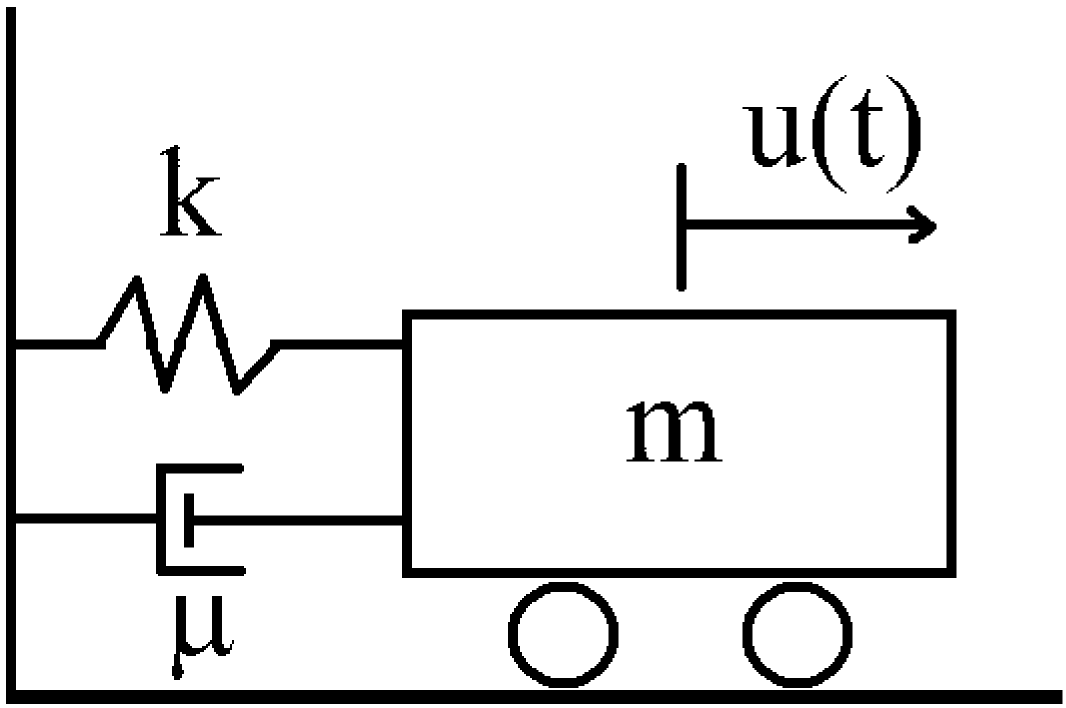
MATEMATINIS MODELIS



$u(t)[m]$ – vežimėlio poslinkis

(nežinomas dydis – priklausomas kintamasis)

$m[kg]$ – vežimėlio masė

$k\left[\frac{N}{m}\right]$ – spyruoklės standis $\left([1N] = \left[\frac{1kg \cdot 1m}{1s^2}\right]\right)$

$\mu\left[\frac{N \cdot s}{m}\right]$ – slopinimo koeficientas

$t[s]$ – laikas (nepriklausomas kintamasis)

PERTVARKIAI

$$m\frac{d^2u}{dt^2} + \mu\frac{du}{dt} + ku = 0,$$

$$u(0) = u_0,\ \frac{du(0)}{dt} = 0$$

Lygties pertvarkiai

$$u(t) = \alpha y(x),\ x = \beta t$$

$$u' = \alpha\beta y'_x,\ \ u'' = \alpha\beta^2 y''_{xx}$$

$$\alpha = u_0,\ y(x) = \frac{u(t)}{u_0}$$

Bedimensinis lygties pavidalas

$$\frac{m\beta^2}{k}y'' + \frac{\mu\beta}{k}y' + y = 0$$

$$y(0) = 1,\ y'(0) = 0$$

MODELIO ANALIZĖ I

$\beta = \sqrt{\frac{k}{m}}$, $\varepsilon = \frac{\mu}{\sqrt{mk}} \ll 1$

$$y'' + \varepsilon y' + y = 0$$

Kai $\varepsilon = 0$, sprendinys:

$$y(x) = Ce^{ix} + \overline{C}e^{-ix} = A\cos x + B\sin x$$

Arba (kadangi $y(0) = 1$, $y'(0) = 0$) $y(x) = \cos x$.

Kai $\varepsilon \neq 0$, turime

$$y(x;\varepsilon) = Ce^{\frac{-\varepsilon + i\sqrt{4-\varepsilon^2}}{2}x} + \overline{C}e^{\frac{-\varepsilon - i\sqrt{4-\varepsilon^2}}{2}x}$$

Atskirasis sprendinys:

$$y(x;\varepsilon) = e^{-\frac{\varepsilon x}{2}} \cos\left(\sqrt{1-\frac{\varepsilon^2}{4}}\,x\right)$$

Mažojo parametro metodas:

$$y(x;\varepsilon) \approx y_0(x) + \varepsilon y_1(x) + \varepsilon^2 y_2(x) + \cdots$$

$$= \cos x + \varepsilon x \cos x + \varepsilon^2 \left(\frac{x \sin x}{8} + \frac{x^2 \cos x}{2}\right) + \cdots$$

Dėl **sekuliariųjų** narių εx, $\varepsilon^2 x^2$, ...
skleidinys **nėra taikytinas** ilgajame kintamojo x kitimo intervale $x \in \left[0, O\left(\frac{1}{\varepsilon}\right)\right]$

Dviejų mastelių metodas:
$\tau = \varepsilon x$ – lėtasis (bedimensinis) laikas
x – (greitasis) laikas (žymėsime t)

Skleidinys:

$$y(t;\varepsilon) = y_0(\tau, t) + \varepsilon y_1(\tau, t) + \varepsilon^2 y_2(\tau, t) + \cdots$$

MODELIO ANALIZĖ II

Atvejis $\beta = \frac{k}{\mu}$, $\varepsilon = \frac{mk}{\mu^2} \ll 1$

$$\varepsilon y'' + y' + y = 0$$

$$y(t;\varepsilon) = Ae^{\frac{-1+\sqrt{1-4\varepsilon}}{2\varepsilon}t} + Be^{\frac{-1-\sqrt{1-4\varepsilon}}{2\varepsilon}t} \approx$$

$$Ae^{-t}e^{\varepsilon t} + Be^{t}e^{-\frac{t}{\varepsilon}}$$

Trys masteliai:
t, $\tau = \varepsilon t$, $T = \frac{t}{\varepsilon}$

Pastebėkime, kad keitinys $y(t;\varepsilon) = Y(T;\varepsilon)$ keičia lygtį:

$$Y'' + Y' + \varepsilon Y = 0, T \gg 1$$